The University of Tokyo
Special Lectures in Information Science Ⅱ
Introduction to Near-Term Quantum Computing

Lesson 14. Utility Scale Experiment III

1. Review of GHZ state

Greenberger–Horne–Zeilinger (GHZ) state is an entangled quantum state that involves at least three subsystems. We review what we have learned about GHZ state in this lecture course.

情報科学科特別講義Ⅱ/量子計算論
Utility Scale Experiment III

2024/07/19
Tamiya Onodera, Kifumi Numata, Toshinari Itoko
IBM Research – Tokyo

Course Schedule 2024 (subject to change)

Date	Lecture Title	Lecturer	Date	Lecture Title	Lecturer
4/5	Invitation to the Utility era	Tamiya Onodera	6/7	Classical simulation (Clifford circuit, tensor network)	Yoshiaki Kawase
4/19	Quantum Gates, Circuits, and Measurements	Kifumi Numata	6/14	Quantum Hardware	Masao Tokunari / Tamiya Onodera
4/26	LOCC (Quantum teleportation/superdense coding/Remote CNOT)	Kifumi Numata	6/21	Quantum circuit optimization (transpilation)	Toshinari Itoko
5/10	Quantum Algorithms: Grover's algorithm	Atsushi Matsuo	6/28	Quantum noise and quantum error mitigation	Toshinari Itoko
5/15 (Wed)	Quantum Algorithms: Phase estimation	Kento Ueda	7/5	Utility Scale Experiment I	Tamiya Onodera
5/24	Quantum Algorithms: Variational Quantum Algorithms (VQA)	Takashi Imamichi	7/12	Utility Scale Experiment II	Yukio Kawashima
5/30 (Thu)	Quantum simulation (Ising model, Heisenberg, XY model), Time evolution (Suzuki Trotter, QDrift)	Yukio Kawashima	7/19	Utility Scale Experiment III	Kifumi Numata / Tamiya Onodera/ Toshinari Itoko

Outline

- A brief lecture on GHZ states [by Tamiya]
    - preparing them on real devices

<Break>

- A jupyter notebook session [by Kifumi]
    - simpler examples
    - your assignment

Greenberger–Horne–Zeilinger state

5 languages

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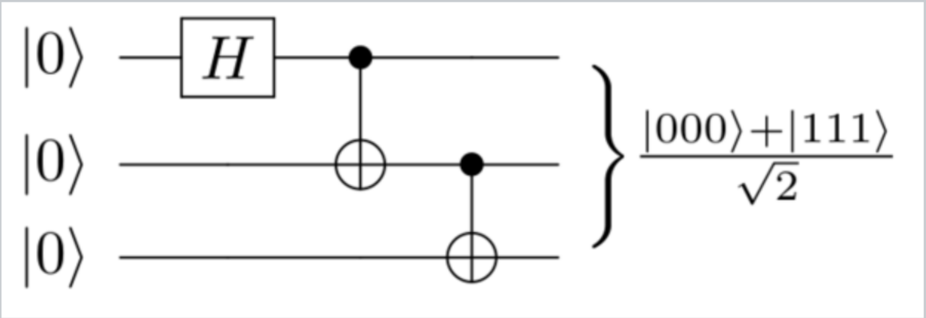
From Wikipedia, the free encyclopedia

In physics, in the area of quantum information theory, a **Greenberger–Horne–Zeilinger state** (**GHZ state**) is a certain type of entangled quantum state that involves at least three subsystems (particle states, qubits, or qudits). The four-particle version was first studied by Daniel Greenberger, Michael Horne and Anton Zeilinger in 1989, and the three-particle version was introduced by N. David Mermin in 1990.



Generation of the 3-qubit GHZ state using quantum logic gates.

https://en.wikipedia.org/wiki/Greenberger%E2%80%93Horne%E2%80%93Zeilinger_state

The Nobel Prize in Physics 2022

Ill. Niklas Elmehed © Nobel Prize Outreach

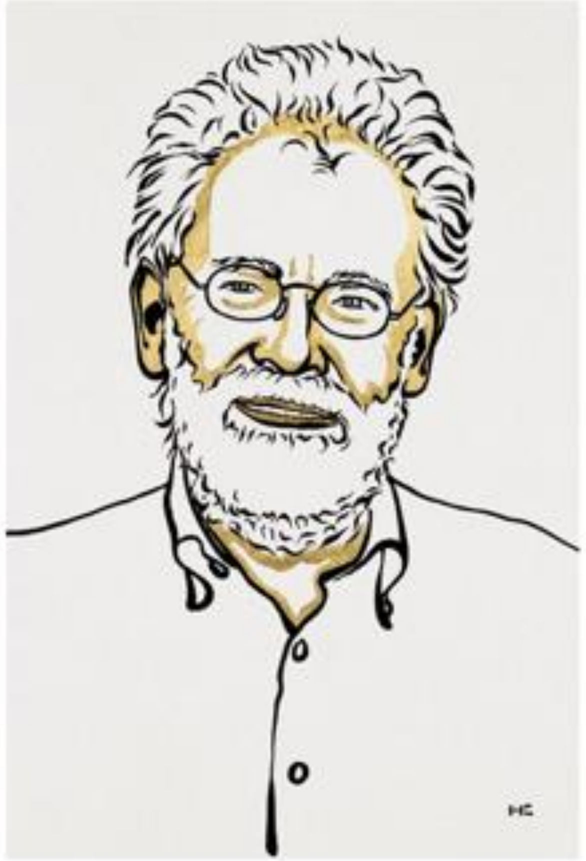
Alain Aspect

Prize share: 1/3

Ill. Niklas Elmehed © Nobel Prize Outreach

John F. Clauser

Prize share: 1/3



Ill. Niklas Elmehed © Nobel Prize Outreach

Anton Zeilinger

Prize share: 1/3

You learnt it on Lecture 2! (Quantum Bits, Gates, and Circuits)

jupyter 20240419_UTokyo Last Checkpoint: 2 months ago

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Markdown JupyterLab Python 3 (ipykernel)

GHZ state

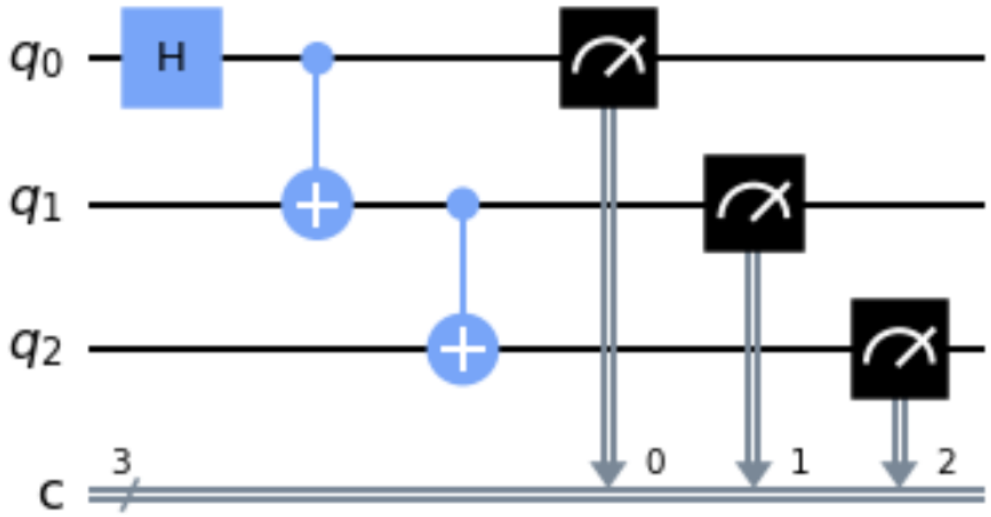
GHZ state (Greenberger-Horne-Zeilinger state) is a maximally entangled state of three or more qubits. GHZ state for three qubits is defined as

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

It can be created with the following quantum circuit.

[32]:
```
qc = QuantumCircuit(3,3)

qc.h(0)
qc.cx(0,1)
qc.cx(1,2)

qc.measure(0, 0)
qc.measure(1, 1)
qc.measure(2, 2)

qc.draw("mpl")
```

[32]:



You learnt it on Lecture 2! (Quantum Bits, Gates, and Circuits)

Exercise 2

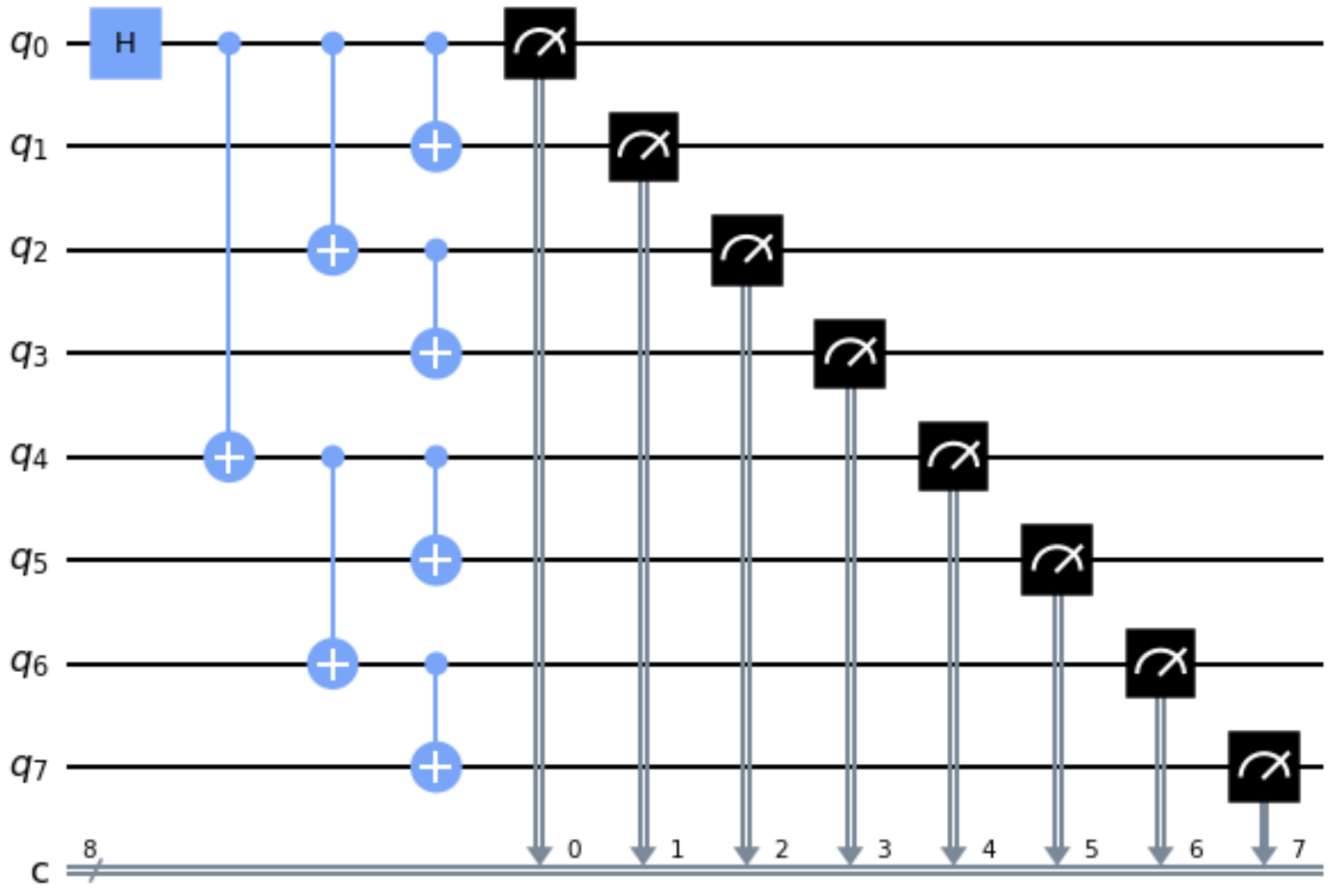
The GHZ state of the 8 quantum bits is as follows

$$\frac{1}{\sqrt{2}}(|00000000\rangle + |11111111\rangle).$$

Let's create this state with the shallowest circuit. The depth of the shallowest quantum circuit is 5 with the measurement gates combined.

[45]:
```
# Step 1
qc = QuantumCircuit(8,8)

##your code goes here##

qc.h(0); qc.cx(0,4)
qc.cx(0,2); qc.cx(4,6)
qc.cx(0,1); qc.cx(2,3); qc.cx(4,5); qc.cx(6,7)

# measure
for i in range(8):
    qc.measure(i, i)

qc.draw("mpl")
#print(qc.depth())
```

[45]:



You learnt more on Lecture 10! (Quantum Circuit Optimization)

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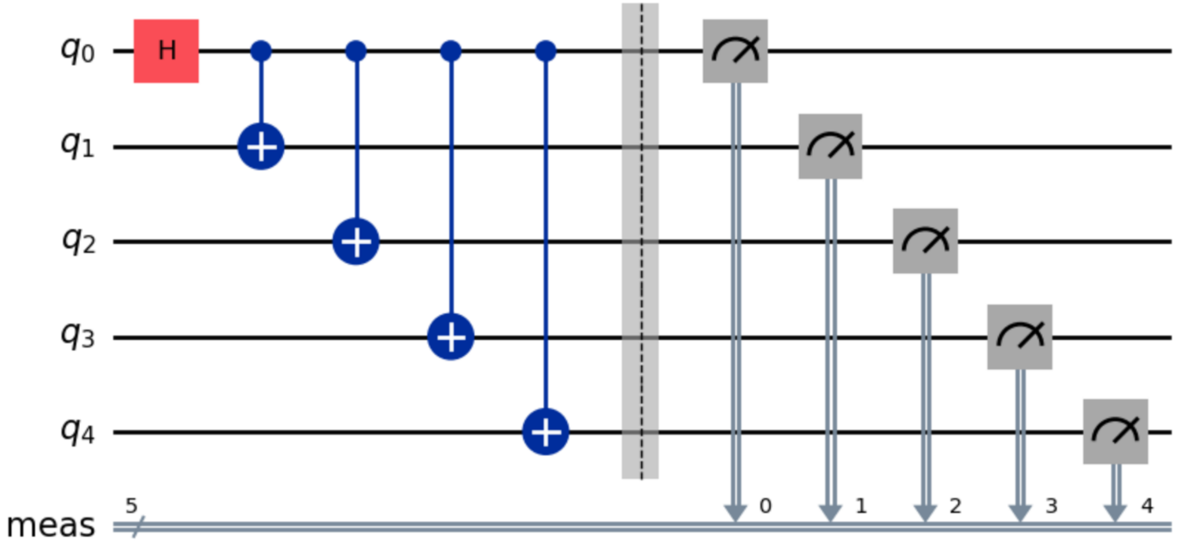
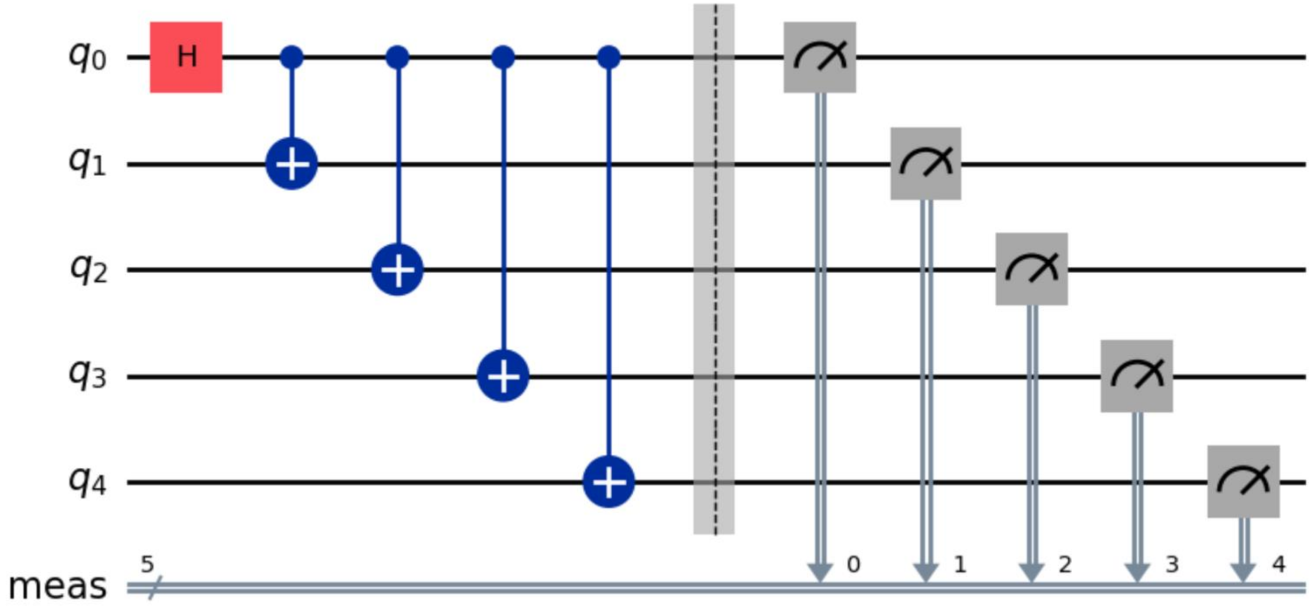
Part 1. Running GHZ circuits with different optimization levels

Circuit optimization matters



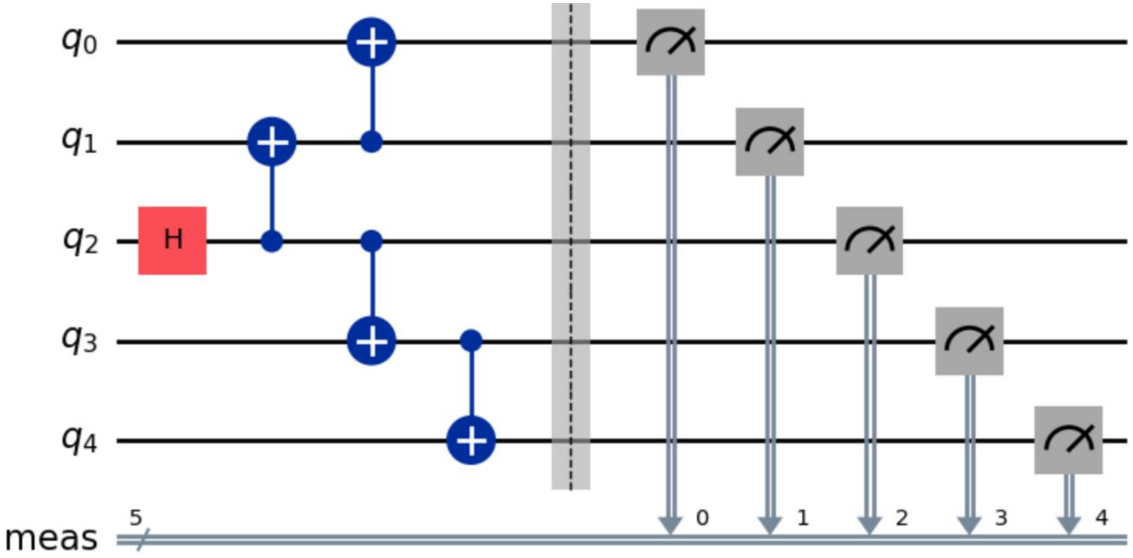



```
for c in [circ0, circ1, circ2]:
    print(c.count_ops())
```

```
OrderedDict({'rz': 77, 'sx': 40, 'ecr': 19, 'measure': 5, 'x': 4, 'barrier': 1})
OrderedDict({'rz': 26, 'sx': 17, 'ecr': 10, 'measure': 5, 'x': 4, 'barrier': 1})
OrderedDict({'rz': 27, 'sx': 13, 'ecr': 7, 'measure': 5, 'x': 1, 'barrier': 1})
```

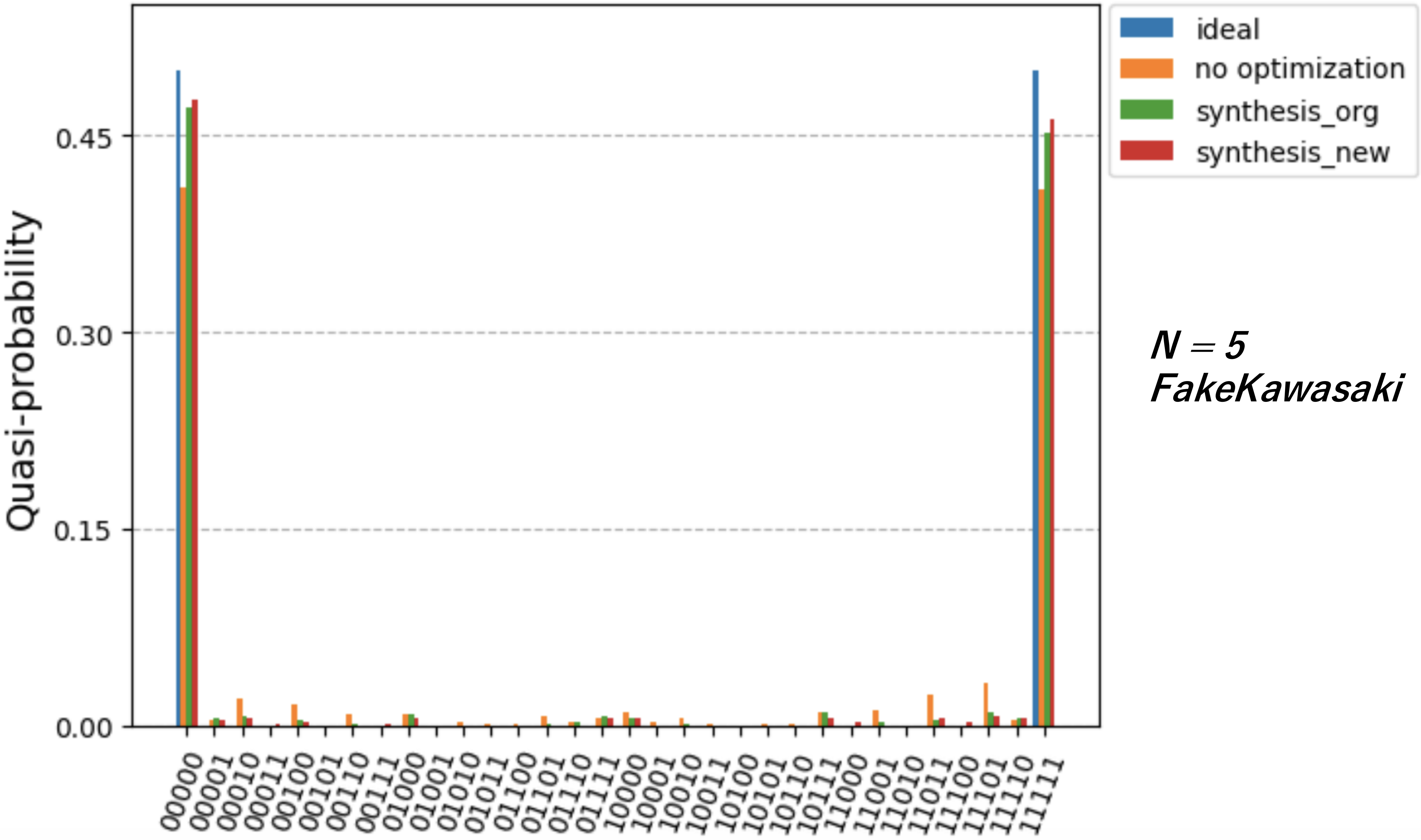
You learnt more on Lecture 10! (Quantum Circuit Optimization)

Circuit synthesis matters




```
for c in [circ_org, circ_new]:
    print(c.count_ops())
```

```
OrderedDict({'rz': 27, 'sx': 13, 'ecr': 7, 'measure': 5, 'x': 1, 'barrier': 1})
OrderedDict({'rz': 19, 'sx': 10, 'measure': 5, 'ecr': 4, 'x': 1, 'barrier': 1})
```



***N* = 5**
FakeKawasaki

2. Why a large GHZ state

How large a GHZ state can be created is a benchmark for a near-term quantum computer. The key points are qubit mapping/routing, circuit depth, and error mitigation/suppression.

Let us prepare a large GHZ sate on a real device!

Why a large GHZ state?

- Many think it serves as a benchmark of a near-term quantum computer.

- Some use it as a benchmark of an algorithm / methodology.
  - e.g., error mitigation

This is an active area of research.

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18-qubit GHZ on a superconducting QC (2020)

PHYSICAL REVIEW A **101**, 032343 (2020)

Editors' Suggestion

Verifying multipartite entangled Greenberger-Horne-Zeilinger states via multiple quantum coherences

Ken X. Wei,[*] Isaac Lauer, Srikanth Srinivasan, Neereja Sundaresan, Douglas T. McClure, David Toyli, David C. McKay, Jay M. Gambetta, and Sarah Sheldon

IBM T.J. Watson Research Center, Yorktown Heights, New York 10598, USA

29-qubit GHZ on a superconducting QC (2022)

PHYSICAL REVIEW A **106**, 012423 (2022)

Efficient quantum readout-error mitigation for sparse measurement outcomes of near-term quantum devices

Bo Yang,[1] Rudy Raymond,[2,3] and Shumpei Uno[3,4]

[1]*Graduate School of Information Science and Technology, The University of Tokyo, Bunkyo-ku, Tokyo 113-8656, Japan*
[2]*IBM Quantum, IBM Research-Tokyo, 19-21 Nihonbashi Hakozaki-cho, Chuo-ku, Tokyo 103-8510, Japan*
[3]*Quantum Computing Center, Keio University, Hiyoshi 3-14-1, Kohoku-ku, Yokohama 223-8522, Japan*
[4]*Mizuho Research & Technologies, Ltd, 2-3 Kanda-Nishikicho, Chiyoda-ku, Tokyo 101-8443, Japan*

27-qubit GHZ on a superconducting QC (2021)

J. Phys. Commun. **5** (2021) 095004 https://doi.org/10.1088/2399-6528/ac1df7

Journal of Physics Communications

PAPER

Generation and verification of 27-qubit Greenberger-Horne-Zeilinger states in a superconducting quantum computer

Gary J Mooney[1], Gregory A L White[1], Charles D Hill[1,2] and Lloyd C L Hollenberg[1]

[1] School of Physics, University of Melbourne, VIC, Parkville, 3010, Australia
[2] School of Mathematics and Statistics, University of Melbourne, VIC, Parkville, 3010, Australia

32-qubit GHZ on an ion-trap QC (2023)

PHYSICAL REVIEW X **13**, 041052 (2023)

Featured in Physics

A Race-Track Trapped-Ion Quantum Processor

S. A. Moses,[1,*,†] C. H. Baldwin,[1,*,‡] M. S. Allman,[1] R. Ancona,[1] L. Ascarrunz,[1] C. Barnes,[1] J. Bartolotta,[1] B. Bjork,[1] P. Blanchard,[1] M. Bohn,[1] J. G. Bohnet,[1] N. C. Brown,[1] N. Q. Burdick,[2] W. C. Burton,[1] S. L. Campbell,[1] J. P. Campora III,[1] C. Carron,[3] J. Chambers,[1] J. W. Chan,[1] Y. H. Chen,[1] A. Chernoguzov,[1] E. Chertkov,[1] J. Colina,[1] J. P. Curtis,[1] R. Daniel,[1] M. DeCross,[1] D. Deen,[3] C. Delaney,[1] J. M. Dreiling,[1] C. T. Ertsgaard,[3] J. Esposito,[1] B. Estey,[1] M. Fabrikant,[1] C. Figgatt,[1] C. Foltz,[1] M. Foss-Feig,[1] D. Francois,[1] J. P. Gaebler,[1] T. M. Gatterman,[1] C. N. Gilbreth,[1] J. Giles,[1] E. Glynn,[1] A. Hall,[1] A. M. Hankin,[1] A. Hansen,[1] D. Hayes,[1] B. Higashi,[3] I. M. Hoffman,[1] B. Horning,[3] J. J. Hout,[1] R. Jacobs,[1] J. Johansen,[1] L. Jones,[1] J. Karcz,[4] T. Klein,[3] P. Lauria,[1] P. Lee,[1] D. Liefer,[1] S. T. Lu,[4] D. Lucchetti,[1] C. Lytle,[1] A. Malm,[1] M. Matheny,[1] B. Mathewson,[1] K. Mayer,[1] D. B. Miller,[1] M. Mills,[1] B. Neyenhuis,[1] L. Nugent,[1] S. Olson,[3] J. Parks,[1] G. N. Price,[1] Z. Price,[1] M. Pugh,[1] A. Ransford,[1] A. P. Reed,[1] C. Roman,[1] M. Rowe,[1] C. Ryan-Anderson,[1] S. Sanders,[1] J. Sedlacek,[2] P. Shevchuk,[1] P. Siegfried,[1] T. Skripka,[1] B. Spaun,[1] R. T. Sprenkle,[1] R. P. Stutz,[1] M. Swallows,[1] R. I. Tobey,[1] A. Tran,[1] T. Tran,[1] E. Vogt,[4] C. Volin,[1] J. Walker,[1] A. M. Zolot,[1] and J. M. Pino[1]

[1]*Quantinuum, 303 South Technology Court, Broomfield, Colorado 80021, USA*
[2]*Quantinuum, 1985 Douglas Drive North, Golden Valley, Minnesota 55422, USA*
[3]*Quantinuum, 12001 State Highway 55, Plymouth, Minnesota 55441, USA*
[4]*Honeywell Aerospace, 12001 State Highway 55, Plymouth, Minnesota 55441, USA*

What matters

- Qubit mapping and routing
- Circuit depth
- Error mitigation / Error suppression

What matters

- Qubit mapping and routing
  - The interaction graph of a circuit should be perfectly embedded into the coupling map of a device.
  - Qubits with lower read-out errors and entangling gates with lower errors should be picked up.
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- Circuit depth
  - A “balanced” tree of entangling gates should be pursued.
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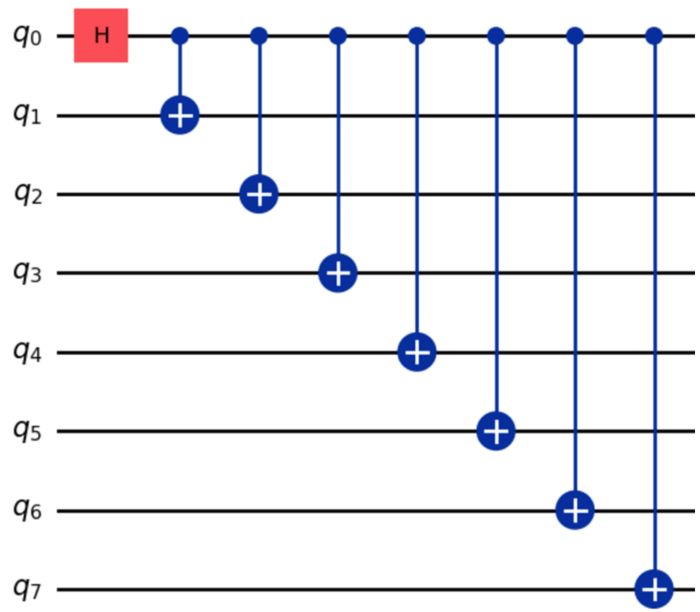
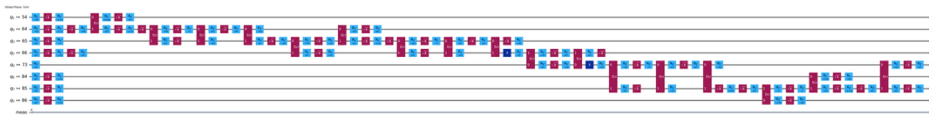
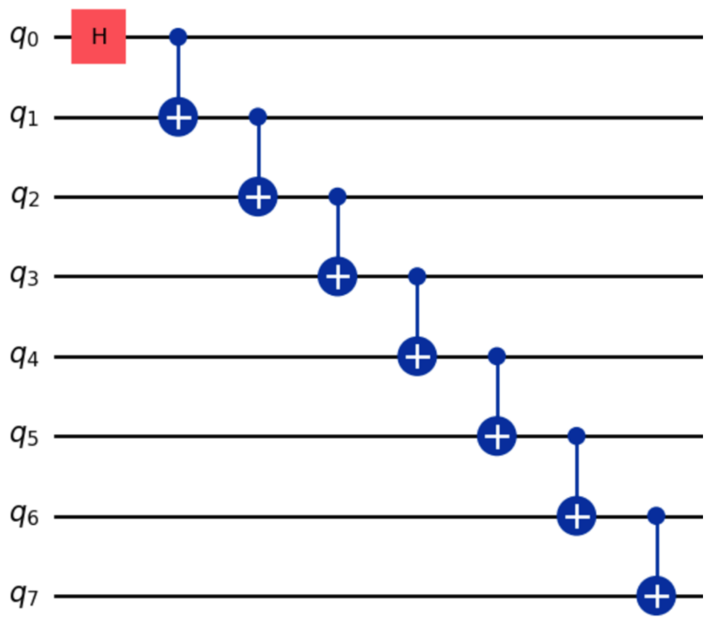
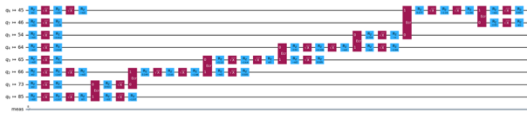


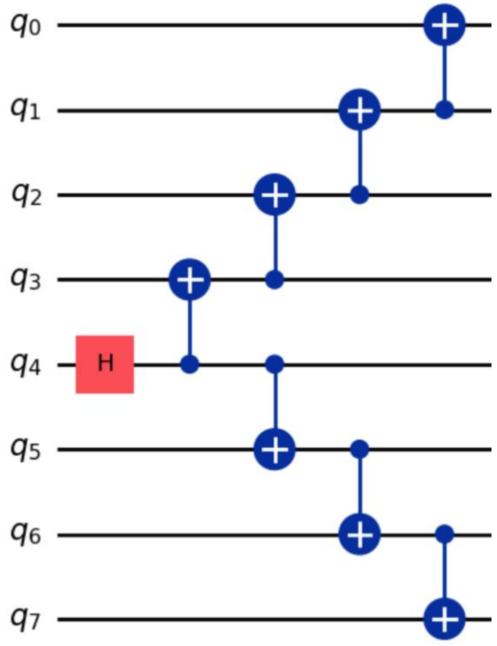


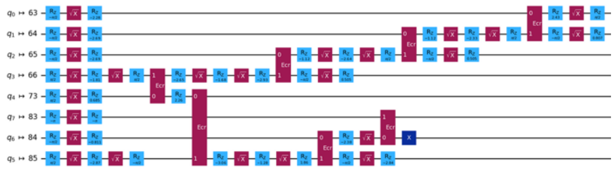
Depth: 76 (two-qubit depth 16)





Depth: 41 (two-qubit depth 7)





Depth: 26 (two-qubit depth 4)

You learnt this on Lecture 9! (Quantum Hardware)

Device map and calibration data

https://quantum.ibm.com/services/resources

ibm_kawasaki OpenQASM 3

Details

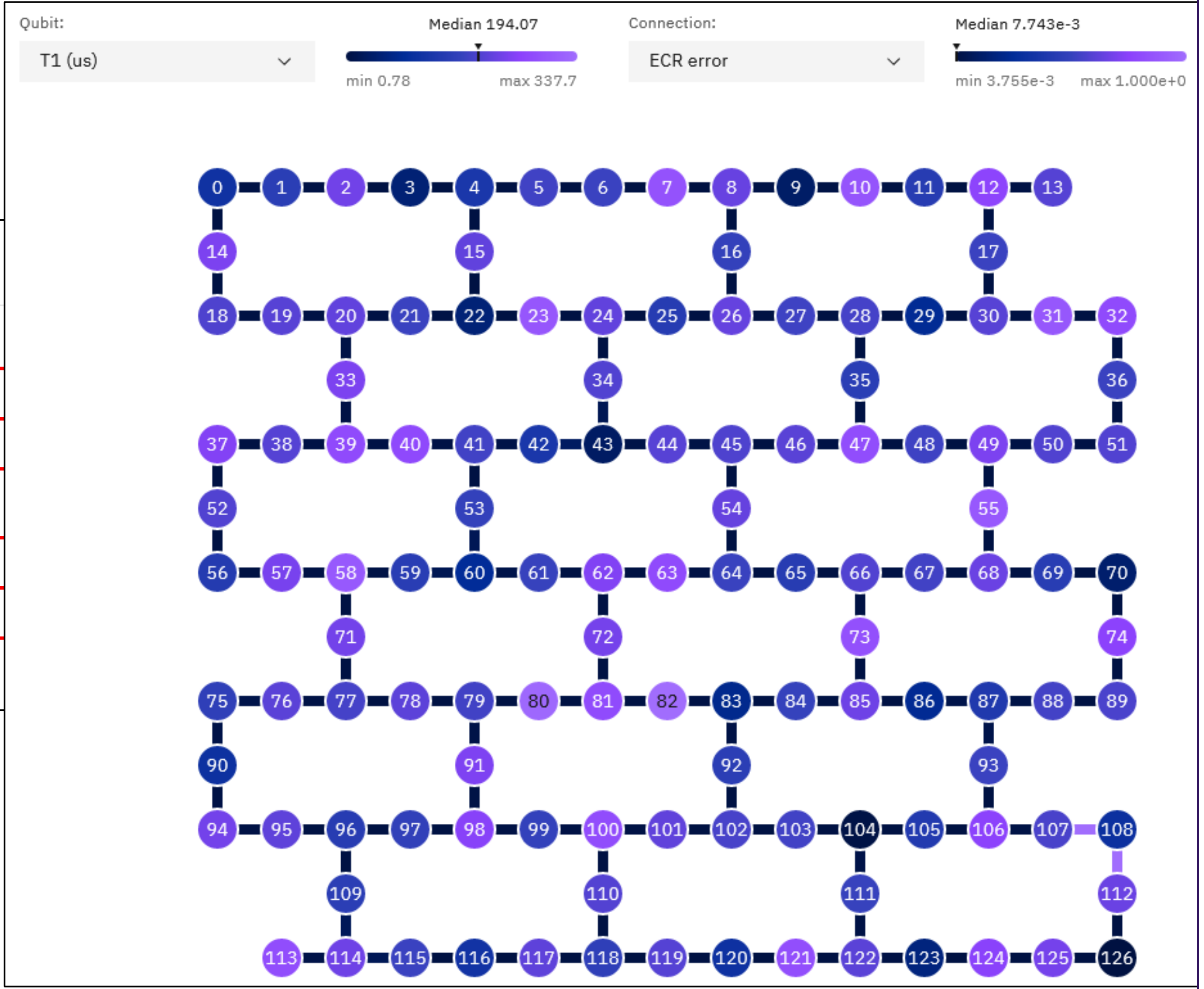
127
Qubits

2.4%
EPLG

5K
CLOPS

Status:	Online
System region:	us-east
Total pending jobs:	210 jobs
Processor type ⓘ:	Eagle r3
Version:	2.1.28
Basis gates:	ECR, ID, RZ, SX, X
Your instance usage:	0 jobs

Median ECR error:	7.653e-3
Median SX error:	2.340e-4
Median readout error:	1.080e-2
Median T1:	183.48 us
Median T2:	138.56 us

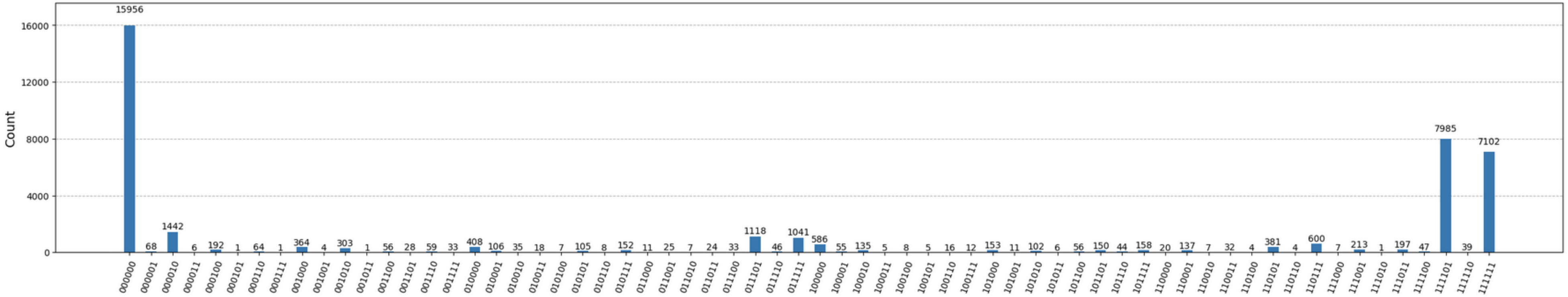


3. How we verify GHZ state?

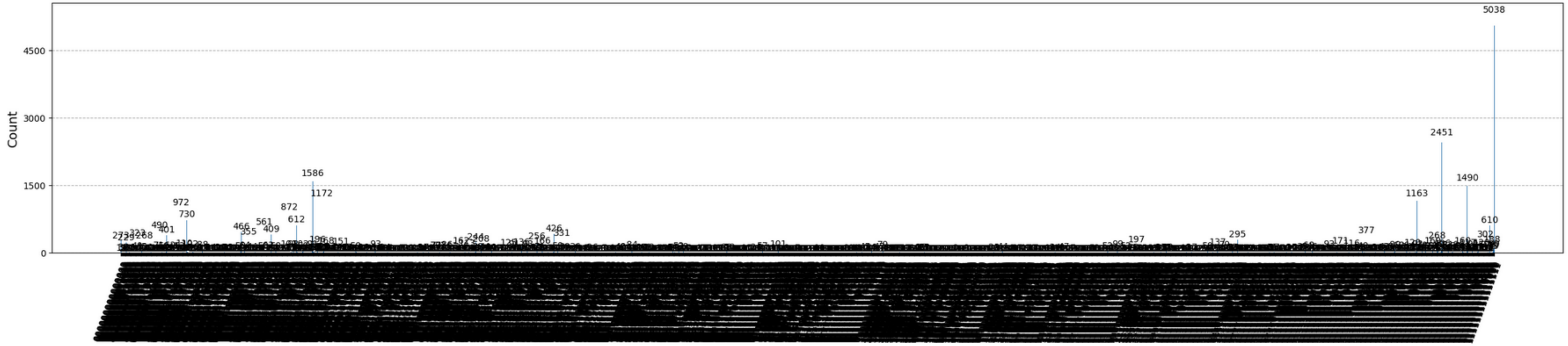
We want to quantify the closeness between what we want to prepare and what we generated on a real device. How we verify it?

How we verify it?

N = 6, ibm_brisbane



N = 12, ibm_brisbane



What matters

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  - The interaction graph of a circuit should be perfectly embedded into the coupling map of a device.
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  - Rely on the transpiler with a “transpiler-friendly” circuit or do it yourself!
- Circuit depth
  - A “balanced” tree of entangling gates should be pursued.
- Error mitigation / Error suppression

How we verify it?

- Want to quantify the closeness between what we want to prepare and what we generated on a real device.
- Different methods proposed.
- We adopt the one based on fidelity in [1].

[1] Otfried Gühne, Chao-Yang Lu, Wei-Bo Gao, and Jian-Wei Pan, "Toolbox for entanglement detection and fidelity estimation", Phys. Rev. A 76, 030305 (2007)

4. Fidelity

We adopt a method to verify GHZ state based on fidelity. Fidelity quantifies the closeness between two quantum states. We learn how to calculate the fidelity using density matrices.

Fidelity

- Quantifies the closeness between two density matrices.
- $F(\rho, \sigma) := \left(tr(\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\right)^2$ for two quantum states ρ and σ.
- $0 \leq F(\rho, \sigma) \leq 1$
- When ρ is a pure state $|\psi\rangle\langle\psi|$, $\quad F(|\psi\rangle\langle\psi|, \sigma) = \langle\psi|\sigma|\psi\rangle = Tr(\sigma|\psi\rangle\langle\psi|)$.

Calculating $F(|\psi\rangle\langle\psi|, \sigma)$

- First, we have $\sqrt{|\psi\rangle\langle\psi|} = |\psi\rangle\langle\psi|$.
- Then, $\sqrt{|\psi\rangle\langle\psi|}\sigma\sqrt{|\psi\rangle\langle\psi|} = |\psi\rangle\langle\psi|\sigma|\psi\rangle\langle\psi| = \langle\psi|\sigma|\psi\rangle|\psi\rangle\langle\psi|$. (Note: $\langle\psi|\sigma|\psi\rangle$ is a scalar.)
- Therefore, $\sqrt{\sqrt{|\psi\rangle\langle\psi|}\sigma\sqrt{|\psi\rangle\langle\psi|}} = \sqrt{\langle\psi|\sigma|\psi\rangle}|\psi\rangle\langle\psi|$
- This leads to $Tr(\sqrt{\langle\psi|\sigma|\psi\rangle}|\psi\rangle\langle\psi|) = \sqrt{\langle\psi|\sigma|\psi\rangle}$.

5. Computing the fidelity in our GHZ experiment

Our original question is whether our GHZ state is good or not. So, we compute the fidelity between the GHZ state which we want to create and the GHZ state which we created on the real device. The measure of $Z^{\otimes N}$ and $Z^{\otimes N}$ after applying unitary transformations gives us that fidelity.

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Computing the Fidelity in our GHZ Experiment.

- What we want to prepare is:

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N})$$

- Let ρ be the state our circuite generated on a real device. Then, what we want to compute is:

$$\begin{aligned} F(|GHZ\rangle\langle GHZ|, \rho) &= Tr(\rho|GHZ\rangle\langle GHZ|) \\ &= \frac{1}{2}\{Tr(\rho|0\rangle\langle 0|^{\otimes N}) + Tr(\rho|1\rangle\langle 1|^{\otimes N}) + Tr(\rho\,(|0\rangle\langle 1|^{\otimes N} + |1\rangle\langle 0|^{\otimes N}))\} \end{aligned}$$

- By repeatedly preparing and measuring ρ with $Z^{\otimes N}$, we can obtain the first two of the traces.

Nice Formula

- We can write $|0\rangle\langle 1|^{\otimes N} + |1\rangle\langle 0|^{\otimes N}$ as

$$\frac{1}{N}\sum_{k=1}^{N}(-1)^k M_k$$

where $M_k = \left(cos(k\pi/N)X + sin(k\pi/N)Y\right)^{\otimes N}$.

- We then have

$$Tr(\rho\,(|0\rangle\langle 1|^{\otimes N} + |1\rangle\langle 0|^{\otimes N})) = \frac{1}{N}\sum_{k=1}^{N}(-1)^k Tr(\rho M_k).$$

- We can thus compute this with N local measurements with $Z^{\otimes N}$ after applying unitary transformations to

$$M_k = \left(Rz(k\pi/N)HZHRz(-k\pi/N)\right)^{\otimes N}.$$

Lesson 14. Utility Scale Experiment III

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6. Verify the formula of unitary transformations

We used unitary transformations to calculate a part of the fidelity. We will verify the formula of that unitary transformations. And then we will go back to what matters to create a large GHZ state.

Verifying the Formula

- First, we have

$$(-1)^k M_k = (-1)^k (exp(-ik\pi/N)|0\rangle\langle 1| + exp(ik\pi/N)|1\rangle\langle 0|)^{\otimes N}.$$

  Expanding the RHS gives 2^N terms each of which is an N-fold tensor product.

- One of the terms is $|0\rangle\langle 1|^{\otimes N}$, whose coefficient is $(-1)^k exp(-ik\pi/N)^N = 1.$ Thus, taking the summation from $k = 1$ to N, the coeffienct of this term ends up with N.

- Similarily, one of the terms is $|1\rangle\langle 0|^{\otimes N}$, whose coefficient is $(-1)^k exp(ik\pi/N)^N = 1.$ Thus, taking the summation from $k = 1$ to N, the coeffienct of this term ends up with N.

- Each of the other terms has the coefficient of

$$(-1)^k exp(-ik\pi/N)^m exp(ik\pi/N)^{N-m} = exp(i2k\pi(1 - m/N))$$

  where $1 \leq m < N$ is the number of $|0\rangle\langle 1|$ in the tensor product. Taking the summation from $k = 1$ to N, the coeffienct of this term ends up with zero.

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- Error mitigation / Error suppression

Break

We then have a Jupyter notebook session.

IBM **Quantum**

Thank you