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1. 次の関係に着目する。

$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle), \quad |1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \tag{1}$$

よって、以下を得る。

$$
\begin{aligned}
|\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \\
&= \alpha\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) + \beta\frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \\
&= \frac{\alpha+\beta}{\sqrt{2}}|+\rangle + \frac{\alpha-\beta}{\sqrt{2}}|-\rangle
\end{aligned} \tag{2}
$$

2.

$$
\begin{aligned}
\mathrm{Prob}\{+1\} &= |\langle 0|\psi\rangle|^2 \\
&= |\langle 0|(\alpha|0\rangle + \beta|1\rangle)|^2 \\
&= |\alpha|^2,
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
\mathrm{Prob}\{-1\} &= |\langle 1|\psi\rangle|^2 \\
&= |\langle 1|(\alpha|0\rangle + \beta|1\rangle)|^2 \\
&= |\beta|^2
\end{aligned} \tag{4}
$$

3.

$$
\begin{aligned}
\mathrm{Prob}\{+1\} &= |\langle +|\psi\rangle|^2 \\
&= |\langle +|(\frac{\alpha+\beta}{\sqrt{2}}|+\rangle + \frac{\alpha-\beta}{\sqrt{2}}|-\rangle)|^2 \\
&= \frac{1}{2}|\alpha+\beta|^2,
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\mathrm{Prob}\{-1\} &= |\langle -|\psi\rangle|^2 \\
&= |\langle -|(\frac{\alpha+\beta}{\sqrt{2}}|+\rangle + \frac{\alpha-\beta}{\sqrt{2}}|-\rangle)|^2 \\
&= \frac{1}{2}|\alpha-\beta|^2,
\end{aligned} \tag{6}
$$

4. 次の関係に着目する。

$$|0\rangle = \frac{1}{\sqrt{2}}(|i\rangle + |-i\rangle), \quad |1\rangle = \frac{1}{\sqrt{2}}(-i|+\rangle + i|-\rangle) \tag{7}$$

よって、以下を得る。

$$
\begin{aligned}
|\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \\
&= \alpha\frac{1}{\sqrt{2}}(|i\rangle + |-i\rangle) + \beta\frac{1}{\sqrt{2}}(-i|i\rangle + i|-i\rangle) \\
&= \frac{\alpha - i\beta}{\sqrt{2}}|i\rangle + \frac{\alpha + i\beta}{\sqrt{2}}|-i\rangle
\end{aligned} \tag{8}
$$

従って、Prob$\{+1\}$、Prob$\{-1\}$ は以下のように計算される。

$$
\begin{aligned}
\mathrm{Prob}\{+1\} &= |\langle i|\psi\rangle|^2 \\
&= |\langle i|(\frac{\alpha - i\beta}{\sqrt{2}}|i\rangle + \frac{\alpha + i\beta}{\sqrt{2}}|-i\rangle)|^2 \\
&= \frac{1}{2}|\alpha - i\beta|^2
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
\mathrm{Prob}\{-1\} &= |\langle -i|\psi\rangle|^2 \\
&= |\langle -i|(\frac{\alpha - i\beta}{\sqrt{2}}|i\rangle + \frac{\alpha + i\beta}{\sqrt{2}}|-i\rangle)|^2 \\
&= \frac{1}{2}|\alpha + i\beta|^2
\end{aligned} \tag{10}
$$