



$$\begin{aligned}
 \rho_B &= \frac{1}{4}(\alpha|0\rangle + \beta|1\rangle)(\alpha^*\langle 0| + \beta^*\langle 1|) \\
 &\quad + \frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)(\alpha^*\langle 0| - \beta^*\langle 1|) \\
 &\quad + \frac{1}{4}(\alpha|1\rangle + \beta|0\rangle)(\alpha^*\langle 1| + \beta^*\langle 0|) \\
 &\quad + \frac{1}{4}(\alpha|1\rangle - \beta|0\rangle)(\alpha^*\langle 1| - \beta^*\langle 0|) \\
 &= \frac{1}{4}(|\alpha|^2|0\rangle\langle 0| + \alpha\beta^*|0\rangle\langle 1| + \beta\alpha^*|1\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|) \\
 &\quad + \frac{1}{4}(|\alpha|^2|0\rangle\langle 0| - \alpha\beta^*|0\rangle\langle 1| - \beta\alpha^*|1\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|) \\
 &\quad + \frac{1}{4}(|\alpha|^2|1\rangle\langle 1| + \alpha\beta^*|1\rangle\langle 0| + \beta\alpha^*|0\rangle\langle 1| + |\beta|^2|0\rangle\langle 0|) \\
 &\quad + \frac{1}{4}(|\alpha|^2|1\rangle\langle 1| - \alpha\beta^*|1\rangle\langle 0| - \beta\alpha^*|0\rangle\langle 1| + |\beta|^2|0\rangle\langle 0|) \\
 &= \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) \\
 &= \frac{1}{2}I \tag{1}
 \end{aligned}$$